EELE 477
Digital Signal Processing

5a

FIR Discrete-Time Systems

Discrete-Time System

- Input and Output are discrete-time sequences:

$$y[n] = F(x[n])$$

- Some systems depend only on the current input:

$$y[n] = 5x[n]$$

$$y[n] = 3(x[n])^2$$

Discrete-Time System (cont.)

- A system may depend on the current and past inputs:

$$y[n] = 5x[n] + 2x[n-1] - 6x[n-2]$$

- Or on past outputs:

$$y[n] = y[n-1] + 2y[n-2] - 3y[n-3]$$

Discrete-Time System (cont.)

- A system can also depend on *future* inputs, but this won’t work in real time:

$$y[n] = 5x[n] + 2x[n+1] - 6x[n+2]$$

- Or any of the infinite combinations and permutations.

Difference Equations

- Many systems can be represented by an input-to-output mathematical expression of the form:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- This is a *difference* equation. It can fully describe the system.

Causal System

- A *causal* system depends only on the current and past inputs.
- A real time system must be causal.
- Non-causal systems may be OK if a time delay is allowable.

Example: Average

$y[n] = 0.5*x[n] + 0.5*x[n-1]$

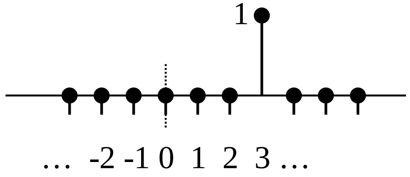
- If x[n] changes slowly, $x[n] \approx x[n-1]$, and y[n] is approximately equal to x[n] .
- If x[n] changes rapidly, $x[n] \neq x[n-1]$, and y[n] is small.

Unit Sample Sequence

- Unit sample, or unit impulse:

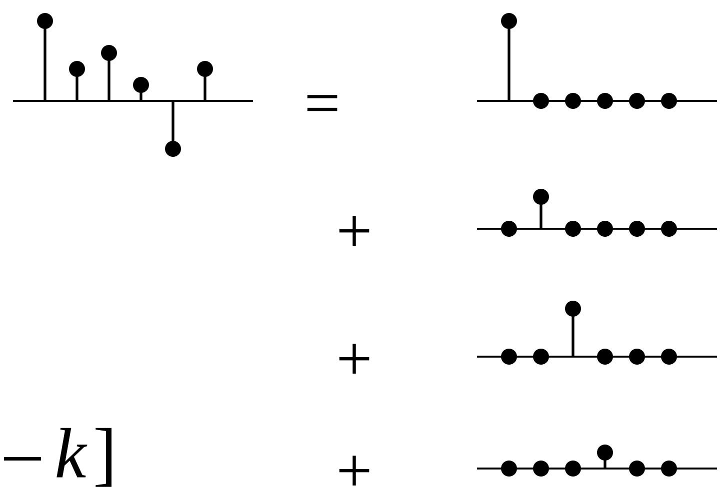
$$\delta[n]=\begin{cases}1, & n=0\\ 0, & else\end{cases}$$

- A shifted impulse: δ[n-3]



Unit Sample Sequence (cont.)

- Notice that a sequence can be represented as a sum of shifted and weighted unit samples:



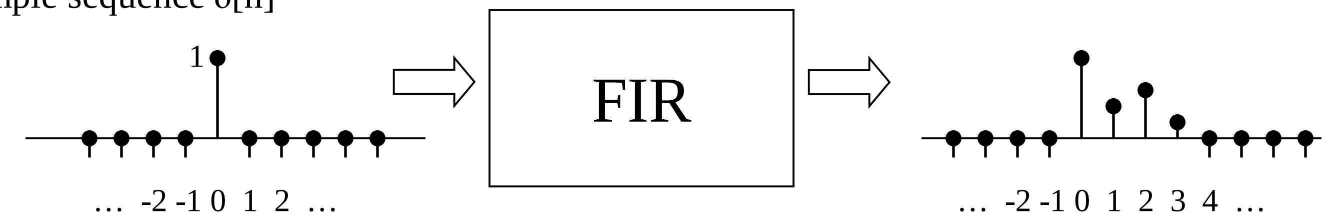
$$x[n] = \sum_k x[k] \cdot \delta[n-k]$$

Finite Impulse Response

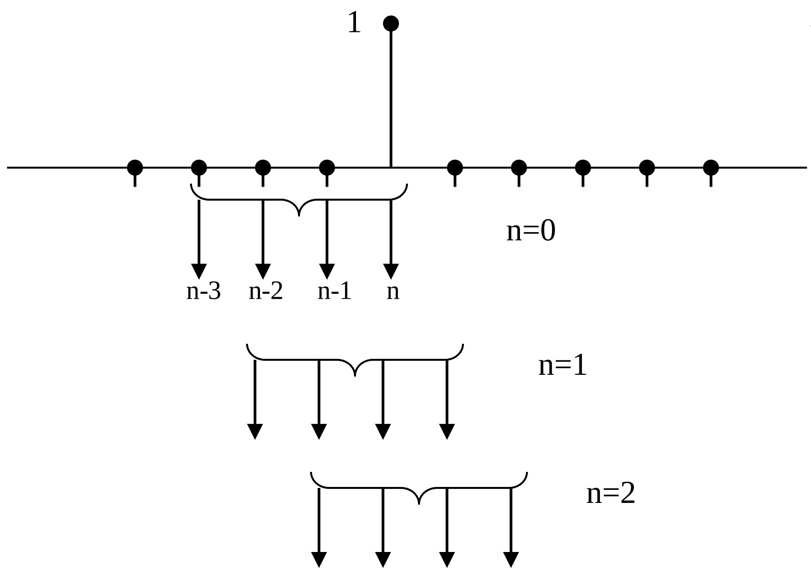
- Discrete-time systems can be characterized by the *impulse response*.
- Apply a single non-zero input sample (a digital *impulse*) and observe the output.
- If the output becomes exactly zero (and stays that way) sometime after the impulse, the system has a *finite-length impulse response*, or is *FIR* for short.

FIR (cont.)

unit sample sequence δ[n]



$$y[n]=a\cdot x[n]+b\cdot x[n-1]+c\cdot x[n-2]+d\cdot x[n-3]$$



$y[0]=a$

$y[1]=b$

$y[2]=c$

$y[3]=d$

For FIR: unit sample response is just the sequence of coefficients

Discrete-Time Convolution

- Can express FIR system as:

$$y[n] = \sum_{k=0}^{M} h[k] \cdot x[n-k]$$

Unit sample response

- For particular *n*, overlap *h* in reverse and sum the products