

DSP First

Laboratory Exercise #6

Filtering Sampled Waveforms

In the experiments of this lab, you will use `firfilt()` to implement filters and begin to understand how the filter's frequency response relates to the action of the filter for smoothing or sharpening. Thus we can characterize how a filter reacts to different frequency components in the input. In addition, you will verify that the cascade connections of LTI systems can be re-ordered without changing the overall frequency response.

1 Overview of Linear Filters

An FIR (finite impulse response) system is described by the formula

$$y[n] = \sum_{k=0}^M b_k x[n - k] \quad (1)$$

Equation (1) gives a formula from which to compute the n^{th} value of the output sequence from values of the input sequence. Recall from Lab 5 that (1) is implemented by the following MATLAB statement:

```
yy = firfilt(bb, xx);
```

where it is assumed that `xx` is a vector of input samples and the vector `b` contains the b_k coefficients of (1) stored in the following way: `[b0, b1, b2, . . . , bM]`. In the experiments of this lab, you will use `firfilt()` to implement filters on several interesting signals.

The frequency response of a general FIR linear time-invariant system is

$$H(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} \quad (2)$$

Recall that you can use the `freqz()` function in MATLAB to compute the frequency response of a discrete-time LTI system. The following MATLAB statements compute and plot the magnitude (absolute value) and phase of the frequency response of a first difference system as a function of $\hat{\omega}$ in the range $-\pi \leq \hat{\omega} \leq \pi$:

```
bb = [1, -1];          %-- Filter Coefficients
omega = -pi:(pi/100):pi;
H = freqz(bb, 1, omega);
subplot(2,1,1);
plot(omega, abs(H))
subplot(2,1,2);
plot(omega, angle(H))
```

For FIR filters of the form (1), the second argument of `freqz(-, 1, -)` must always be equal to 1.



2 Warm-up

The instructor verification sheet is included at the end of this lab.

To begin, start MATLAB and then load the data for this lab as follows:

```
load lab6dat
```

This loads a data file `lab6dat.mat` containing several filters and signals. The variables in this data file are:



`x1`: a stair-step signal such as one might find in one sampled scan line from a TV test pattern image.

`xtv`: an actual scan line from a digital image.

`x2`: a speech waveform sampled at 8000 samples/second.

`h1`: the coefficients for a FIR discrete-time filter of the form of (1).

`h2`: a second FIR filter.

You will use these in the following experiments.

2.1 Properties of Discrete-Time Filters

- (a) The frequency responses of discrete-time filters are *always* periodic with period equal to 2π . Explain why this is the case by stating a definition of the frequency response and then considering two input sinusoids whose frequencies are $\hat{\omega}$ and $\hat{\omega} + 2\pi$.

$$x_1[n] = e^{j\hat{\omega}n} \quad \text{versus} \quad x_2[n] = e^{j(\hat{\omega} + 2\pi)n}$$

Prove that the outputs from (1) will be identical.

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- (b) When several systems are cascaded (connected so that the output of one is the input to the next one) it is possible to calculate the overall frequency response by multiplying the individual frequency responses. Since the frequency response for a general FIR linear time-invariant system given by (2) can be considered a polynomial in the variable $e^{j\hat{\omega}}$, multiplying two frequency responses amounts to nothing more than multiplying two polynomials.

As a quick review, multiply the following two polynomials by hand:

$$x + 0.5x^2 - 2x^3 \quad \text{and} \quad 1 + x - 0.25x^3 \quad (3)$$

- (c) Now use the MATLAB command `firfilt` to convolve the two sequences whose elements are the coefficients of the polynomials in part (b). Remember to use a coefficient of 0 for those powers of x that are missing in the polynomial. How do your results compare with the coefficients of the multiplied polynomial of the previous step? Read the `help` on `firfilt` which mentions that convolving two vectors of polynomial coefficients is equivalent to multiplying the polynomials.
- (d) Use polynomial multiplication to solve for the overall frequency response of a cascade of a 3-point averager and a 5-point averager. Hint: Use polynomial multiplication on the coefficients then evaluate the result using `freqz`).

Instructor Verification (separate page)

3 Lab: Sampling and Filters

The upcoming experiments in this lab will demonstrate several important facts about LTI filtering of sampled signals. You should be on the lookout for the following:

- that filters of the form of (1) can modify the frequency spectrum of any signal (not just complex exponentials) in interesting ways.
- that the order of cascaded LTI systems does not affect the overall response.
- that lowpass filters “smooth” a signal and highpass filters “roughen” a signal, so listening tests on filtered speech give sounds that are either “muffled” or “crisper.”

3.1 Filtering a Stair-Step Signal

In this experiment we are going to investigate the two systems shown in Figs. 1 and 2. In these two systems, the system called “First Difference” is defined by the difference equation

$$y[n] = x[n] - x[n - 1] \quad (4)$$

and the system called “5-Point Averager” is defined by the equation:

$$y[n] = \frac{1}{5} \sum_{k=0}^4 x[n - k] \quad (5)$$

The first test signal used will be a stair-step signal in which the signal is constant for different intervals, but the constant value of the signal in each flat region is different.

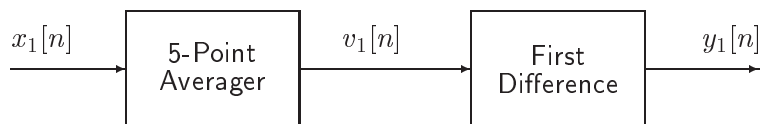


Figure 1: First cascade system: averaging operator followed by first difference.

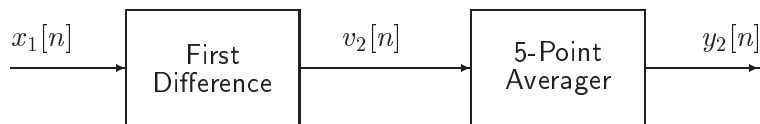


Figure 2: Second cascade system: differencing operator followed by averaging.

3.2 Implementation of 5-Point Averager

Use the MATLAB function `firfilt()`, which can be found in the *DSP First Toolbox*, to implement the 5-Point Averager, i.e., compute `v1`.



CD-ROM

`firfilt.m`

- (a) Plot $\mathbf{x1}$ and $\mathbf{v1}$ in the same figure window using a two-panel `subplot`. The signals have different lengths, but the plot should start at the same index, i.e., $n = 0$.
Give a qualitative description of how the 5-Point Averager system changed the input signal. Estimate the time shift between the input and output signals. Express this delay in number of samples.
- (b) Use `freqz()` to compute the frequency response of the 5-Point Averager and plot its magnitude as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$. From the shape of the frequency response curve, determine which frequency region is “passed” by the filter? Which frequencies are rejected? Relate the frequency response to the qualitative description of the time-domain response.
- (c) Plot the phase response versus frequency. Measure the slope of the phase response and compare this slope to the time shift between the input and output signals.

3.3 Implementation of First Difference System

- (a) Plot $\mathbf{x1}$ and $\mathbf{v2}$ in the same figure window using a two-panel `subplot`.
- (b) Describe qualitatively how the First Difference system changed the input signal? Where are the peaks of $\mathbf{v2}$? What is the nature of the input $\mathbf{x1}$ at the places where the peaks of $\mathbf{v2}$ occur?
- (c) Use `freqz()` to compute the frequency response of the First Difference and plot its magnitude as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$. Is the shape of the frequency response curve consistent with your interpretation of what the system did to the input signal? Is the first difference a high-pass or low-pass filter?

3.4 Implementation of First Cascade (Fig. 1)

- (a) Use the MATLAB function `firfilt()` to implement the overall system of Fig. 1 by first computing the output of the 5-Point Averager, $\mathbf{v1}$, and then using $\mathbf{v1}$ as input to the First Difference system in Fig. 1 to compute the output $\mathbf{y1}$.
- (b) Use `freqz()` to compute the frequency response of the cascaded system and plot its magnitude and phase as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$.

3.5 Implementation of Second Cascade (Fig. 2)

- (a) Use the MATLAB function `firfilt()` to implement the overall system of Fig. 2 by first computing the output of the First Difference, $\mathbf{v2}$, and then using $\mathbf{v2}$ as input to the 5-Point Averager system in Fig. 2 to compute the output $\mathbf{y2}$.
- (b) Use `freqz()` to compute the frequency response of the cascaded system and plot its magnitude and phase as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$. Compare to the overall frequency response computed for Fig. 1.

3.6 Comparison of Systems of Figs. 1 and 2

Execute the MATLAB statement `sum((y1-y2).*(y1-y2))`. Write the mathematical expression that is being evaluated by MATLAB. Discuss how this calculation measures the error between the two alternate implementations and then discuss the implications of the result.

3.7 Filtering the Speech Waveform



A sampled speech waveform is stored in the variable `x2` in the file `lab6dat.mat`. Two sets of filter coefficients are stored in `h1` and `h2` (i.e., these are the “ b_k s” for two different filters). Use `length` to find out how many filter coefficients are contained in `h1` and `h2`. In this experiment we will test these filters on the speech signal.

- (a) Filter the speech signal with filter `h1` using the statements

```
y1 = firfilt(h1, x2);
inout(x2, y1, 3000, 1000, 3)
```



The M-file `inout()` will plot two very long signals together on the same plot. It formats the plot so that the input signal occupies the first, third, and fifth lines, etc. while the output is on the second, fourth, and sixth lines etc. Type `help inout` to find out more.

Compare the input and output signals. Is the output “rougher” or “smoother” than the input signal?

- (b) Use `freqz()` to plot the frequency response of the system defined by the coefficients `h1` as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$. Why is `h1` called a “lowpass filter”?
- (c) Since the vector of filter coefficients is rather long, a `stem` plot of `h1` can be informative to show the nature of the filter. Use the `stem` plot to find a point of symmetry in the coefficients.
- (d) Filter the speech signal with filter `h2` and plot the input and output using the statements

```
y2 = firfilt(h2,x2);
inout(x2, y2, 3000, 1000, 3)
```

Compare the input and output and state whether the output is “rougher” or “smoother” than the input signal.

- (e) Use `freqz()` to plot the frequency response of the system defined by the coefficients `h2` as a function of frequency for $-\pi \leq \hat{\omega} \leq \pi$. Why is `h2` called a “highpass filter”?
- (f) Make a `stem` plot of `h2` and look for a symmetry in the coefficients.
- (g) Make an “A-B-C” listening comparison by executing the following statements¹

```
sound([x2; y1; y2], 8000)
```

Comment on your perception of the filtered outputs versus the original

- (h) What do you expect to hear when you execute the following statement? Why?

```
sound([x2; (y1+y2)], 8000)
```

Was your expectation confirmed? If you added the two frequency responses together

$$H_1(e^{j\hat{\omega}}) + H_2(e^{j\hat{\omega}})$$

what would you expect the answer to be?

¹In MATLAB version 5, use `soundsc`,

Lab 6

Instructor Verification Sheet

Staple this page to the end of your Lab Report.

Name: _____ Date: _____

Part 2.1 Explain the periodicity of the frequency response of a discrete system:

Verified: _____

Part 2.1 Demonstrate polynomial multiplication/discrete convolution:

Verified: _____